SPECTRAL PROPERTIES OF WIGNER RANDOM MATRICES

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The entries of Wigner random matrices are, up to the symmetry constraints, independent and identically distributed random variables. In this minicourse, I am going to discuss recent results about the spectrum of Wigner matrices in the limit where the size N of the matrices tends to infinity.

The first rigorous result in the study of Wigner matrices was obtained in 1955 by Wigner, who proved the convergence of the density of states (the density of the eigenvalues) to the famous semicircle law. Wigner's original result concerned macroscopic intervals containing order N eigenvalues. In the first lecture, I plan to show how Wigner's original result can be extended to show convergence to the semicircle law also on smaller intervals containing typically order one (in N) eigenvalues.

In the second lecture, I will present some consequences of the refined local semicircle law. First of all, I will show the complete delocalization of the eigenvectors of Wigner matrices (every component of any eigenvector is typically of the same size). Afterwards, I will show a Wegner-type estimate (an upper bound for the probability to find an eigenvalue in a small interval, proportional to the size of the interval) and I will prove upper bounds on the probability to find k eigenvalues in an arbitrarily small interval; as a consequence, the property of level repulsion will follow.

In the last two lectures, I will discuss the universality of the local eigenvalue statistics. Universality refers to the fact that, in the limit of large N, the (local) eigenvalue correlation functions depend on the symmetry of the ensemble but are otherwise independentof the choice of the probability law of the matrix entries. I will first show how universality can be inferred (for hermitian ensembles) by extending an approach introduced by Johansson and combining it with the local semicircle law and with a time reversal idea. Then I will discuss a different approach to prove universality for ensembles with arbitrary symmetry, based on the introduction of the so-called local relaxation flow.