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Stable Lvy processes in a cone

Banuelos and Bogdan (2004) and Bogdan, Palmowski and Wang (2016) analyse the asymptotic tail distribution of the first time a stable (Lvy) process in dimension $d \geq 2$ exits a cone. We use these results to develop the notion of a stable process conditioned to remain in a cone as well as the notion of a stable process conditioned to absorb continuously at the apex of a cone (without leaving the cone). As self-similar Markov processes we examine some of their fundamental properties through the lens of its Lamperti-Kiu decomposition. In particular we are interested to understand the underlying structure of the Markov additive process that drives such processes. As a consequence of our interrogation of the underlying MAP, we are able to provide an answer by example to the open question: If the modulator of a MAP has a stationary distribution, under what conditions does its ascending ladder MAP have a stationary distribution?

We show how the two forms of conditioning are dual to one another. Moreover, we construct the null-recurrent extension of the stable process killed on exiting a cone, showing that it again remains in the class of self-similar Markov processes. In the spirit of several very recent works, the results presented here show that many previously unknown results of stable processes, which have long since been understood for Brownian motion, or are easily proved for Brownian motion, become accessible by appealing to the notion of the stable process as a self-similar Markov process, in addition to its special status as a Lvy processes with a semi-tractable potential analysis.