Fluctuation theory for spectrally positive additive Lévy fields

A spectrally positive additive Lévy field is a multidimensional field obtained as the sum $\mathbf{X}_{\mathrm{t}} = \mathrm{X}^{(1)}_{t_1} + \mathrm{X}^{(2)}_{t_2} + \dots + \mathrm{X}^{(d)}_{t_d}$, $\mathrm{t} = (t_1, \dots, t_d) \in \mathbb{R}^d_+$, where $\mathrm{X}^{(j)} = {}^t(X^{1,j}, \dots, X^{d,j})$, $j = 1, \dots, d$, are d independent $\mathbb{R}^d$-valued Lévy processes issued from 0, such that $X^{i,j}$ is non decreasing for $i \neq j$ and $X^{j,j}$ is spectrally positive. It can also be expressed as $\mathbf{X}_{\mathrm{t}} = \mathbb{X}_{\mathrm{t}}\mathbf{1}$, where $\mathbf{1} = {}^t(1, 1, \dots, 1)$ and $\mathbb{X}_{\mathrm{t}} = (X^{i,j}_{t_j})_{1 \le i,j \le d}$. The main interest of spaLf's lies in the Lamperti representation of multitype continuous time branching processes. In this work, we study the law of the first passage times $\mathbf{T}_{\mathrm{r}}$ of such fields at levels $-\mathrm{r}$, where $\mathrm{r} \in \mathbb{R}^d_+$. We prove that the field $\{(\mathbf{T}_{\mathrm{r}}, \mathbb{X}_{\mathbf{T}_{\mathrm{r}}}), \mathrm{r} \in \mathbb{R}^d_+\}$ has stationary and independent increments and we describe its law in terms of this of the spaLf $\mathbf{X}$. In particular, the characteristic exponent of $(\mathbf{T}_{\mathrm{r}}, \mathbb{X}_{\mathbf{T}_{\mathrm{r}}})$ solves a functional equation leaded by the characteristic exponent of $\mathbf{X}$. Then we derive an extension of Kemperman's formula for spectrally positive Lévy processes which connects the distribution of $\{(\mathbf{T}_{\mathrm{r}}, \mathbb{X}_{\mathbf{T}_{\mathrm{r}}}), \mathrm{r} \in \mathbb{R}^d_+\}$ to this of $\{\mathbb{X}_{\mathrm{t}}, \mathrm{t} \in \mathbb{R}^d_+\}$. (This is a joint work with Marine Marolleau – University of Angers.)

Loïc Chaumont – LAREMA – UMR CNRS 6093, Université d'Angers, 2 bd Lavoisier, 49045 Angers cedex 01

E-mail address: `loic.chaumont@univ-angers.fr`