Limit theorems for additive functionals of continuous-time random walk

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Outline

1. Limit theorems for additive functionals of CTRW
2. CTRW with location-dependent intensity
3. CTRW in a Poisson shot-noise potential environment
4. Open questions

Continuous-time random walk

CTRW $X = \{X_t, t \geq 0\}$:

- iid jumps ξ_n, $n \geq 1$;
- iid inter-jump times θ_n, $n \geq 1$.

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Exponential $\theta_n \Rightarrow$ CTRW = compound Poisson process.

Otherwise, non-Markov.

Question

For a function $f : \mathbb{R} \to \mathbb{R}$ what is the behavior, as $t \to \infty$, of the additive functional

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Discrete- or continuous-time ergodic Markov process with invariant probability measure ν

- LLN:

$$\frac{A_t}{t} \to \nu(f) := \int f(x)\nu(dx), t \to \infty;$$

- CLT:

$$\frac{A_t - \nu(f)}{\sqrt{t}} \xrightarrow{d} \mathcal{N}(0, \sigma_f^2).$$

For a (discrete-time, non-lattice) centered random walk X in the domain of attraction of α-stable law with $\alpha > 1$, under suitable normalization,

$$c_n \sum_{k=1}^{[nt]} f(S_k) \xrightarrow{d} \int_{-\infty}^{\infty} f(x)dx \cdot \ell_\alpha(0,t),$$

where $\ell_\alpha(t,0)$ is the local time at zero of α-stable Levy motion on $[0,t]$.

Skorokhod and Slobodenjuk 1970, Borodin and Ibragimov 1994, Jeganathan 2004.

Contents

A1. Jump sizes ξ_n are centered and belong to the domain of attraction to α-stable law with $\alpha \in (1, 2]$. Then,

$$\left\{ \frac{1}{L(n) n^{1/\alpha}} \sum_{k=1}^{[nt]} \xi_k, t \geq 0 \right\} \xrightarrow{d} \{ Z_\alpha(t), t \geq 0 \}$$

towards an α-stable Lévy motion $Z_\alpha(t)$.

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A3. The times θ_n between jumps are integrable:

$$\mathrm{E}\,[\,\theta_1\,] = \mu.$$

Set $c_n = L(n) n^{1/\alpha - 1}$.

Theorem

Let X be a CTRW. Under assumptions A1–A3,

$$c_n \int_0^{nt} f(X_s) ds, \quad t \geq 0,$$

converge as $n \to +\infty$ in fdd to

$$\mu^{1/\alpha} \int_{-\infty}^{\infty} f(x) dx \cdot \ell_\alpha(t, 0), \quad t \geq 0,$$

where $\ell_\alpha(t, 0)$ is the local time at zero of Z_α on $[0, t]$.

Proof

Write

$$c_n \int_0^n f(X_s)ds = c_n \sum_{k=1}^{N_n} \theta_k f(S_{k-1}) + c_n\big(n - \tau_{N_n}\big) f(S_{N_n}).$$

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Say, $f(x) = \mathbb{1}_{[a,b]}(x)$: by LLN,

$$c_m \sum_{k=1}^m \theta_k \mathbb{1}_{[a,b]}(S_{k-1}) = c_m \sum_{k \le m : S_{k-1} \in [a,b]} \theta_k$$

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Write same for any f, then apply results for RW S_k (Jeganathan 2004).

Lattice case

Assume A1, A3, replace A2 by

$$P(\xi_1 \in \{a + b\mathbb{Z}\}) = 1$$

with some $a \in \mathbb{R}, b > 0$.
If $\sum_{k=-\infty}^{\infty} |f(a + bk)| < \infty$, then

$$\left\{ c_n \int_0^{nt} f(X_s)ds, t \geq 0 \right\} \xrightarrow{fdd} \left\{ \mu^{1/\alpha} b \sum_{k=-\infty}^{\infty} f(a + bk) \cdot \ell_\alpha(t, 0), t \geq 0 \right\}.$$

Contents

Let the time between jumps depend on the location of the walker: the intensity of jumps from a location x is $\Lambda(x) > 0$.

Markovian case: a pure jump process with the generator

$$(A\psi)(x) = \Lambda(x) \int_{\mathbb{R}} \big(\psi(x-y) - \psi(x)\big) F_{\xi_1}(dy).$$

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Let us call it Λ-*delayed CTRW*.

Crucial ingredients of the proof:

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Distribution of S_i “converges” to “ergodic distribution of S”, which is “uniform distribution on $\mathbb{R}$”. So probably

$$\frac{1}{n} \sum_{i=1}^{n} h(S_i) \to \overline{h},$$

where $\overline{h} = \lim_{T\to+\infty} \frac{1}{2T} \int_{-T}^{T} h(x)dx$ is the Cesaro average of h.

B1. Jump size are centered and their distribution belongs to the normal domain of attraction to α-stable law with $\alpha \in (1, 2)$: i.e. $L(n) = \sigma > 0$;

$$\left\{ \frac{1}{\sigma n^{1/\alpha}} \sum_{k=1}^{[nt]} \xi_k, t \geq 0 \right\} \xrightarrow{d} \{ Z_\alpha(t), t \geq 0 \} .$$

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$$\int_{-\infty}^{\infty} x^2 \left| f_{\xi_1}(x) - f_{Z_\alpha(1)}(x) \right| dx < \infty.$$

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B4. There exists $\overline{\Lambda^{-1}} > 0$ such that for some $r > \alpha$,

$$\sup_{|x| \leq t^r} \left| \frac{1}{t} \int_x^{x+t} \Lambda(y)^{-1} dy - \overline{\Lambda^{-1}} \right| \to 0, t \to +\infty.$$

Proposition

Under A3, B1–B4,

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\theta_i}{\Lambda(S_i)} \xrightarrow{P} \mu\,\overline{\Lambda^{-1}}, n \to \infty,$$

and

$$\frac{N_t}{t} \xrightarrow{P} \frac{1}{\mu\,\overline{\Lambda^{-1}}}, t \to \infty.$$

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Proof: Markov approximation and the results on convergence of W-functionals (Kulik 2006, Kartashov and Kulik 2009).

Theorem

Let X be a Λ-delayed CTRW. Under assumptions A2–A3 *on* $f(x) = \frac{g(x)}{\Lambda(x)}$ *and* B1–B4,

$$\sigma n^{1/\alpha - 1} \int_0^{nt} g(X_s) ds, \quad n \geq 0,$$

converges as $n \to +\infty$ in fdd to

$$\mu^{1/\alpha} \cdot \left(\overline{\Lambda^{-1}}\right)^{1/\alpha - 1} \cdot \int_{-\infty}^{\infty} \frac{g(x)}{\Lambda(x)} dx \cdot \ell_\alpha(t, 0), \quad u \geq 0.$$

Contents

Consider a *random* Λ–so-called Poisson shot-noise potential:

$$\Lambda(x,\gamma) = e^{-\sum_{y\in\gamma}\phi(x-y)} =: e^{-E_\phi(x,\gamma)};$$

$\phi\colon \mathbb{R}\to[0,\infty)$, γ is a homogeneous Poisson configuration.
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Sufficient condition for Λ to be well defined: $\phi\in L^1(\mathbb{R})$. We need a bit stronger:

C1. $\phi\in C(\mathbb{R})$ and there exist some $C,\beta>0$ such that

$$|\phi(x)| \le \frac{C}{1+|x|^{1+\beta}}.$$

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Then (Carmona and Molchanov 1995)

$$\sup_{|x|\le n} \left|E_\phi(x,\gamma)\right| = O\left(\frac{\log|n|}{\log\log|n|}\right), n \to \infty.$$

Proposition

Under the assumption C1, *for any* $\delta > 0$,

$$\sup_{|x|\le n} \Lambda(x,\gamma)^{-1} = o(n^\delta), n \to \infty,$$

a.s. and for any $r > 1$,

$$\sup_{|x|\le t^r} \left| \frac{1}{t} \int_x^{x+t} \Lambda(y,\gamma)^{-1} dy - \mathsf{E}\left[\Lambda(0,\gamma)^{-1} \right] \right| \to 0, t \to +\infty,$$

almost surely.

Lemma

Let $\{Y_t, t \in [0,T]\}$ *be a centered measurable process which is* a*-independent for some* $a \in (0,T)$*, i.e.* $\{Y_t, t \in A\}$ *and* $\{Y_t, t \in B\}$ *are independent whenever* $\inf_{t\in A, s\in B} |t-s| \geq a$*. For each integer* $k \geq 1$*, there exists a universal constant* $C_k > 0$ *such that*

$$\mathrm{E}\left[\left(\int_0^T Y_t dt\right)^{2k}\right] \leq C_k (aT)^k \sup_{t\in[0,T]} \mathrm{E}\left[Y_t^{2k}\right].$$

C2. Either $g \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ and the characteristic function φ_{ξ_1} of jump sizes is integrable to some power $p > 0$, or $g \in L^1(\mathbb{R})$ and there exist some $C, \varepsilon > 0$ such that $|g(x)| \leq C(1 + |x|^{\varepsilon})^{-1}$ for all $x \in \mathbb{R}$.

Theorem

Let X be a CTRW delayed by Poisson shot noise potential Λ independent of X. Under A3, B1, B2, C1, C2*,*

$$\sigma n^{1/\alpha - 1} \int_0^{nt} g(X_s) ds, \quad t \geq 0,$$

converges as $n \to +\infty$ in fdd to

$$\mu^{1/\alpha} \cdot \exp\left\{ \left(\frac{1}{\alpha} - 1 \right) \int_{-\infty}^{\infty} \left(e^{\phi(y)} - 1 \right) dy \right\} \cdot \int_{-\infty}^{\infty} \frac{g(x)}{\Lambda(x, \gamma)} dx \cdot \ell_\alpha(t, 0),$$

with ℓ_α independent of Λ.

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