MATHEMATICAL RESULTS ON THE HUBBARD MODEL

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The Hubbard Model is the simplest non-trivial many-fermion model. For N electrons on a finite lattice Λ, $\#\Lambda < \infty$, its Hamiltonian is given by

$$H \; = \; \sum_{i=1}^{N} h_i + \frac{U}{2} \sum_{i \neq j} V(x_i - x_j),$$

where $U > 0$ is a number, h_i is the kinetic energy (hopping matrix) $(t_{x,y})_{x,y \in \Lambda}$ of the i^{th} particle, and $V(x-y) = \delta_{x,y}$ is the on-site repulsion.

In spite of its simple form, the phase diagram of the Hubbard model has a rich structure which is expected to bear the correct microscopic description of magnetic ordering and high-T_c superconductivity. So, not surprisingly, the Hubbard model is the work horse of condensed matter theory, and the theoretical physics literature on it is abundant.

In four lectures I will focus on the following particular aspects:

1. Introduction of the Hubbard model, simple mathematical properties, heuristic discussion of its phase diagram.
2. Some mathematical results: exact solution in 1-d, Lieb's theorems on the total spin, Nagaoka's ferromagnetism...
3. The Hartree-Fock approximation for the Hubbard model: antiferromagnetism at half-filling, ferromagnetism at low filling and for strong coupling.
4. *If time permits:* Construction of KMS states for sufficiently high (in fact: not too small) temperature.