

Hilbert's Third Problem and Beyond

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In the nineteenth century mathematicians were wondering how the elementary notions of area and volume relate to the corresponding modern concepts, the latter being based on infinitesimal calculus. For the planar case, the Wallace-Bolyai-Gerwien Theorem implies that the elementary area of any polygon (defined by chopping the polygon into polygonal parts and reassembling those) is coinciding with modern notions of area. Hilbert doubted that an analogue result in three-dimensional space holds for the volume and stated this as his third problem at the International Congress of Mathematicians at Paris in 1900.

We will explain the two-dimensional setting in detail, motivate Hilbert's question and discuss Max Dehn's solution. We will also sketch Dehn's proof, which makes use of an additive functional on the family of all polyhedra. Since these early developments, additive functionals have been shown useful both in pure mathematics and in applications. A milestone for this progress was Hadwiger's characterization of Minkowski functionals. The last part of my talk is devoted to sketching this result and some related newer developments.