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Analysis of Some Greedy Algorithms for the Single-Sink Fixed-Charge Transportation Problem

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Abstract

The single-sink fixed-charge transportation problem (SSFCTP) consists in finding a minimum cost flow from a number of nodes to a single sink. Beside a cost proportional to the amount shipped, the flow cost encompass a fixed charge. The SSFCTP is an important subproblem of the well-known fixed-charge transportation problem. Nevertheless, just a few methods for solving this problem have been proposed in the literature. In this paper, some greedy heuristic solutions methods for the SSFCTP are investigated. It is shown that two greedy approaches for the SSFCTP known from the literature can be arbitrarily bad, whereas an approximation algorithm proposed in the literature for the binary min-knapsack problem has a guaranteed worst case bound if adapted accordingly to the case of the SSFCTP.

Key words: fixed-charge transportation problem, min-knapsack problem, greedy algorithms, worst-case analysis, approximation algorithms

1 Introduction

The single-sink fixed-charge transportation problem (SSFCTP) is to decide on the amounts $x_j \geq 0$ of shipments to be made from a given set of suppliers $j = 1, \ldots, n$ to a single sink in such a way that the suppliers' capacities b_j are respected and the sink's demand D is satisfied at minimum shipment cost. The cost of shipping $x_j > 0$

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units from a supplier j to the sink involves a fixed charge f_j as well as costs $c_j x_j$ that are proportional to the quantity shipped. The mathematical formulation of the problem is:

$$Z = \min \sum_{j=1}^{n} \Big(c_j x_j + f_j y_j \Big) \tag{1}$$

$$\text{s.t.:} \sum_{j=1}^{n} x_j = D \,, \tag{2}$$

$$0 \leq x_j \leq b_j y_j \quad \text{for } j = 1, \ldots, n \,, \tag{3}$$

$$y_j \in \{0, 1\} \quad \text{for } j = 1, \ldots, n \,, \tag{4}$$

where $c_j \geq 0$, $f_j \geq 0$, and $0 < b_j \leq D$ for $j = 1, \ldots, n$ can be assumed without loss of generality.

The SSFCTP is an important relaxation of the fixed-charge transportation problem, which is in turn a special fixed-charge network flow problem, a class of optimization problems that have a large number of applications in the area of supply chain planning as well as logistics and telecommunications network design. Herer et al. (1996) mention a number of applications of the SSFCTP in the fields of supplier selection, product distribution and fleet selection as well as process selection. Chauhan and Proth (2003) introduce a heuristic procedure for solving the "concave cost supply problem" that generalizes the SSFCTP by assuming concave transportation costs and requiring a minimum quantity to be delivered by a selected supplier. Exact solution methods for the SSFCTP based on implicit enumeration, branch-and-bound and dynamic programming techniques were considered by Herer et al. (1996), Alidaee and Kochenberger (2005) and Klose (2006).

In this paper, heuristic solution methods for the SSFCTP are investigated. The SSFCTP is closely related to the binary min-knapsack problem (min-KP). In case of zero or constant unit transportation costs $c_j = 0$ for $j = 1, \ldots, n$, the SSFCTP reduces to a min-KP. Firstly, this shows that the SSFCTP is NP-hard and can be solved to optimality in pseudo-polynomial time by means of dynamic programming. Secondly, this suggests that heuristic solution algorithms for the SSFCTP with a guaranteed worst-case performance should be obtained by accordingly adjusting corresponding methods introduced in the literature for the min-KP. This is done in Sect. 4, where it is shown that an approximation method proposed by Csirik et al. (1991) for the min-KP also gives a 2-approximation algorithm for the SSFCTP. In a similar way as Csirik et al. (1991) did for the min-KP, the method can be further improved in order to obtain a reduced worst-case ratio of 1.5. Beforehand, however, we briefly analyze the LP relaxation of the SSFCTP in Sect. 2 and show in Sect. 3 that two simple greedy algorithms known from the literature on the SSFCTP might produce arbitrarily bad solutions. Sect. 5 gives then a computational comparison of the various heuristic methods, and Sect. 6 concludes the paper with a summary of the findings.

2 Linear programming relaxation

Since the fixed costs f_j are non-negative, it is optimal to keep each of the variables y_j as small as possible if the integrality requirements are dropped. Hence, one may set $y_j = x_j/b_j$ and reduce the LP relaxation of the SSFCTP to the linear program

$$\begin{aligned} Z^{LP} = \min \ & \sum_{j=1}^{n} e_j x_j \\ \text{s.t.: } & \sum_{j=1}^{n} x_j = D\,, \\ & 0 \le x_j \le b_j \text{ for } j = 1, \dots, n\,, \end{aligned} \tag{5}$$

where $e_j := c_j + f_j/b_j$. Similar to the linear relaxation of the binary knapsack problem, the above linear program can easily be solved as follows by sorting the relative costs e_j.

Proposition 1. *Assume that $e_1 \le e_2 \le \dots \le e_n$ and let $s \in \{1, \dots, n\}$ be such that*

$$\sum_{j=1}^{s-1} b_j < D \quad \textit{and} \quad \sum_{j=1}^{s} b_j \ge D\,. \tag{6}$$

An optimal solution to the linear program (5) is then given by

$$x_j = \begin{cases} b_j, & \textit{for } j = 1, \dots, s-1 \\ D - \sum_{j=1}^{s-1} b_j, & \textit{for } j = s \\ 0, & \textit{for } j = s+1, \dots, n\,. \end{cases} \tag{7}$$

Proof. Let σ and η_j $(j = 1, \dots, n)$ be dual multipliers of the demand and upper bound constraints, respectively. The dual

$$\begin{aligned} Z^{LP} = \max \ & \sigma D - \sum_{j=1}^{n} \eta_j b_j \\ \text{s.t.: } & \sigma - \eta_j \le e_j \text{ for } j = 1, \dots, n\,, \\ & \eta_j \ge 0 \qquad \text{for } j = 1, \dots, n \end{aligned}$$

of the linear program (5) has the feasible solution $\eta_j = \max\{0, e_s - e_j\}$ for $j = 1, \dots, n$ and $\sigma = e_s$, whose objective function value

$$\begin{aligned} e_s D - \sum_{j=1}^{n} \max\{0, e_s - e_j\} b_j &= e_s D - \sum_{j=1}^{s-1} (e_s - e_j) b_j \\ &= \sum_{j=1}^{s-1} e_j b_j + e_s \Big(D - \sum_{j=1}^{s-1} b_j \Big) \end{aligned}$$

equals that of the primal solution (7). □

Solving the LP relaxation by means of sorting the suppliers can be done in $O(n \log n)$ time. Using similar procedures as those known for the knapsack problem (Balas and Zemel, 1980), it is possible to determine the "split supplier" s in linear time $O(n)$.

As in the case of the min-KP, the LP bound Z^{LP} can be arbitrarily bad. The following proposition further exemplifies this property.

Proposition 2. *There exist instances of the SSFCTP such that Z^{LP}/Z is arbitrarily close to zero.*

Proof. Consider the following series of problem instances with $n = 2$, $0 \le c_1 \le c_2$, $b_1 = b_2 = b > 0$, $D = b+\epsilon^2$, $0 \le f_1 \le 1$, $f_2 = 1/\epsilon$, and $0 < \epsilon < 1$. If $\epsilon > 0$ approaches zero, the LP bound

$$Z^{LP} = f_1 + c_1 b + c_2\epsilon^2 + \epsilon/b$$

converges to the finite value $f_1 + c_1 b$, whereas the optimal cost

$$Z = f_1 + c_1 b + c_2\epsilon^2 + 1/\epsilon$$

diverges to infinity. $\square$

For the maximization version of the knapsack problem it is known that the upper bound provided by the LP relaxation is no greater than two times the optimal solution value (see, e.g, Kellerer et al. (2004), p. 19). It is easy to see that the same worst-case performance also holds for the LP bound of the following maximization version of the SSFCTP.

$$z^* = \max \sum_{j=1}^{n} \Big(c_j x_j - f_j y_j \Big) \tag{8}$$

$$\text{s.t.: } \sum_{j=1}^{n} x_j \le D\,, \tag{9}$$

$$0 \le x_j \le b_j y_j \quad \text{for } j = 1, \ldots, n\,, \tag{10}$$

$$y_j \in \{0, 1\} \quad \text{for } j = 1, \ldots, n\,, \tag{11}$$

where $c_j \ge 0$, $f_j \ge 0$, and $0 < b_j \le D$ for $j = 1, \ldots, n$.

Proposition 3. *Let z^{LP} denote the solution value to the LP relaxation of the program (8)–(11). Then $z^{LP} \le 2z^*$ and there exist instances of the problem such that $z^{LP} \ge 2z^* - \epsilon$ for every arbitrarily small $\epsilon > 0$.*

Proof. The proof is analogous to the one showing the corresponding property in case of the binary knapsack problem (see, e.g., Kellerer et al. (2004), p. 19). Define $p_j = c_j - f_j/b_j$ and assume that $p_1 \ge p_2 \ge \cdots \ge p_n$. If some of the p_j are negative, let k be the smallest index with $p_k < 0$. If no such k exists, set $k = n + 1$. If $\sum_{j=1}^{k-1} b_j \le D$, then $z^* = z^{LP}$ results. Otherwise let $s \in \{1, \ldots, k-1\}$ be such that

(6) holds. The solution value of the LP relaxation is then given by

$$z^{LP} = P_{s-1} + p_s x_s^{LP}, \text{ where } P_{s-1} = \sum_{j=1}^{s-1} p_j b_j \text{ and } x_s^{LP} = D - \sum_{j=1}^{s-1} b_j \,. \tag{12}$$

The two solutions

$$\begin{aligned} & (x_1, \ldots, x_{s-1}, x_s, \ldots, x_n) = (b_1, \ldots, b_{s-1}, 0, \ldots, 0) \\ \text{and} \quad & (x_1, \ldots, x_{s-1}, x_s, x_{s+1}, \ldots, x_n) = (0, \ldots, 0, b_s, 0, \ldots, 0) \end{aligned}$$

are feasible. Hence

$$z^{LP} = P_{s-1} + p_s x_s^{LP} \leq P_{s-1} + p_s b_s \leq 2z^*$$

follows. Furthermore, for the series of instances with $n = 2$, $b_1 = b_2 =: b > 0$, $D = 2b - \epsilon$, $0 < \epsilon < b$, $c_1 = c_2 =: c > 0$ and $c(b - \epsilon) < f_1 = f_2 =: f$, one obtains $z^* = pb$ and $z^{LP} = pb + p(b - \epsilon) = 2pb - p\epsilon$, where $p := p_1 = p_2 = c - f/b$. $\square$

3 Popular greedy algorithms

The greedy algorithm for the binary knapsack problem consists in filling the knapsack with the most profitable items until the knapsack's capacity is reached. Profitability is thereby measured as the ratio of profit to an item's weight. A straightforward application of this greedy method to the SSFCTP is then to select the solution to the LP relaxation as a feasible solution to the SSFCTP. This solution's total cost is then given by

$$\begin{aligned} Z^G &= \sum_{j=1}^{s-1} e_j b_j + f_s + c_s x_s^{LP} = Z^{LP} + f_s + (c_s - e_s) x_s^{LP} \\ &= Z^{LP} + f_s \Big(1 - x_s^{LP}/b_s\Big) \,. \end{aligned} \tag{13}$$

In case of the min-KP, the greedy solution can be arbitrarily bad (see, e.g., Kellerer et al. (2004), p. 403). Since the min-KP is a special case of the SSFCTP, this is also true for the greedy solution (13).

Proposition 4. *There exist instances of the SSFCTP such that Z^G/Z is not bounded from above.*

Proof. The following series of instances slightly modifies a problem instance of the min-KP used by Kellerer et al. (2004) for the purposes of showing that the greedy procedure for the min-KP might be arbitrarily bad: $n = 3$, $D \geq 4$ and

$$\begin{array}{lll} c_1 = \frac{1}{D-1}\,, & c_2 = 1\,, & c_3 = 2\,, \\ f_1 = 1\,, & f_2 = D - 2\,, & f_3 = 1\,, \\ b_1 = D - 1\,, & b_2 = D - 1\,, & b_3 = 1\,. \end{array}$$

Because $e_1 = 2/(D-1) \le e_2 = 1 + (D-2)/(D-1) \le e_3 = 3$, the greedy solution is $x_1 = D-1$, $x_2 = 1$, $x_3 = 0$ and has objective function value $Z^G = D+1$. The optimal solution of value $Z = 5$ is $x_1 = D-1$, $x_2 = 0$, and $x_3 = 1$. Hence the ratio Z^G/Z diverges to infinity with increasing D. □

In case of the binary knapsack problem, the so-called "Ext-greedy" procedure gives a 2-approximation method. Ext-greedy simply compares the greedy solution's objective value with the highest profit of a single item and selects the better solution. It seems plausible that a similar property also holds for the maximization version (8)–(11) of the SSFCTP. The greedy solution value z^G for this problem is given by

$$z^G = \sum_{j=1}^{s-1} p_j b_j + \max\Big\{0,\, c_s x_s^{LP} - f_s\Big\}\,, \text{where } x_s^{LP} = D - \sum_{j=1}^{s-1} b_j\,.$$

The value p_j and the index s are defined as in Prop. 3. The Ext-greedy solution's objective value z^{eG} is then

$$z^{eG} = \max\Big\{z^G, \max\{p_j b_j : j = 1, \dots, n\}\Big\}\,. \tag{14}$$

Proposition 5. *In case of the maximization version* (8)–(11) *of the SSFCTP, the Ext-greedy algorithm has relative performance guarantee of* $\frac{1}{2}$, *that is* $z^{eG}/z^* \ge \frac{1}{2}$.

Proof. From relaxation and (14) we conclude that

$$z^* \le z^{LP} = \sum_{j=1}^{s-1} p_j b_j + p_s x_s^{LP} \le z^G + p_s b_s \le z^{eG} + z^{eG} = 2z^{eG}\,.$$

To show that this bound is tight consider the following example that slightly modifies an instance used by Kellerer et al. (2004) for the purposes of analyzing Ext-greedy for the knapsack problem: $n = 3$, $b_1 = 1$, $b_2 = b_3 = M$, $D = 2M$, $c_1 = 3$, $f_1 = 1$, $c_2 = c_3 = M+1$ and $f_2 = f_3 = M^2$. The relative profits are then $p_1 = 2 > p_2 = p_3 = 1$. Because

$$c_3(D - b_1 - b_2) - f_3 = (M+1)(M-1) - M^2 = -1 < 0\,,$$

the greedy solution of total profit $z^G = 2+M$ is $x_1 = 1$, $x_2 = M$ and $x_3 = 0$. In case of this example, the Ext-greedy solution equals the greedy solution. The optimal solution of total profit $z^* = 2M$ is to set $x_1 = 0$, $x_2 = x_3 = M$. Hence the ratio $z^{eG}/z^* = (2+M)/(2M)$ converges to $\frac{1}{2}$ with increasing value of M. □

Herer et al. (1996) introduce a variation of the greedy procedure for the SSFCTP and Chauhan et al. (2004) apply an accordingly modified version of this method to the "linear cost supply problem", which slightly generalizes the SSFCTP by including variable upper bounds $x_j \ge l_j y_j$ on the flow variables. After the greedy procedure sets the first $s-1$ flow variables x_j to the upper bounds b_j, the remaining demand $\overline{D} := D - \sum_{j=1}^{s-1} b_j$ is usually smaller than the split supplier's capacity. It is thus reasonable to use the "effective capacity" $\overline{b}_j := \min\{\overline{D}, b_j\}$ for the purposes of linearizing the fixed costs and selecting the next supplier to which a positive amount of supply should be allotted. This yields the heuristic solution procedure below that we call "adaptive" greedy procedure.

Step 1: For $j = 1, \ldots, s-1$ set $x_j = b_j$. Set $\overline{D} = D - \sum_{j=1}^{s-1} b_j$.

Step 2: Find supplier i with

$$c_i + \frac{f_i}{\min\{\overline{D}, b_i\}} = \min_{j=s,\ldots,n} \left\{ c_j + \frac{f_j}{\min\{\overline{D}, b_j\}} : x_j = 0 \right\}.$$

Step 3: Set $x_i = \min\{\overline{D}, b_i\}$ and $\overline{D} := \overline{D} - x_i$. If $\overline{D} > 0$ return to Step 2.

The procedure's computational effort amounts to $O(n^2)$ in contrast to a time complexity of $O(n \log n)$ or even $O(n)$ for performing the greedy procedure. Chauhan et al. (2004) claim that this adaptive greedy procedure gives a 2-approximation algorithm for the SSFCTP. The following proposition shows that this is not the case; the algorithm may also produce arbitrarily bad solutions.

Proposition 6. *There exist instances of the SSFCTP such that Z^{AG}/Z is not bounded from above, where Z^{AG} denotes the solution value obtained with the adaptive greedy procedure.*

Proof. Consider the following series of problem instances with $n \geq 5$, $D = 2^{n-1}$, $c_j = 0$ for $j = 1, \ldots, n$, and

$$\begin{aligned} f_1 &= 1\,, & b_1 &= 2^{n-2} + 1\,, \\ f_j &= 2^{n-2}\,, & b_j &= 2^{j-1} - 1 \text{ for } j = 2, \ldots, n-2\,, \\ f_{n-1} &= 2^{n-1}\,, & b_{n-1} &= 2^{n-2} - 1\,, \\ f_n &= n-3\,, & b_n &= n-3\,. \end{aligned}$$

For these data the following inequality holds:

$$\begin{aligned} b_n + \sum_{j=2}^{n-2} b_j &= n - 3 + \sum_{j=2}^{n-2} (2^{j-1} - 1) \\ &= (n-3) - (n-3) + \sum_{j=1}^{n-3} 2^j \\ &= 2^{n-2} - 2 \leq b_{n-1} \leq b_1 < D\,. \end{aligned}$$

Furthermore, $2^{n-2} - 2 + b_1 = 2^{n-1} - 1 < D$ and $2^{n-2} - 2 + b_{n-1} < D$. Feasibility thus requires that suppliers 1 and $n-1$ supply a positive amount. The optimal solution is therefore $x_1 = 2^{n-2} + 1$, $x_j = 0$ for $j \in \{2, \ldots, n-2, n\}$ and $x_{n-1} = 2^{n-2} - 1$. The solution's objective value is $Z = 2^{n-1} + 1$.

In the first two iterations, the adaptive greedy procedures selects $x_1 = b_1$ and $x_n = b_n = n-3$. The remaining demand then amounts to

$$D_3 = D - b_1 - b_n = 2^{n-1} - 2^{n-2} - 1 - n + 3 = 2^{n-2} - n + 2\,.$$

Because $b_{n-1} = 2^{n-2} - 1 > D_3 = 2^{n-2} - n + 2$ for $n \geq 4$, one obtains

$$\begin{aligned}\frac{f_{n-1}}{\min\{D_3, b_{n-1}\}} &= \frac{f_{n-1}}{D_3} = \frac{2^{n-1}}{2^{n-2} - n + 2} \\ &> \frac{f_{n-2}}{\min\{D_3, b_{n-2}\}} = \frac{f_{n-2}}{b_{n-2}} = \frac{2^{n-2}}{2^{n-3} - 1} = \frac{2^{n-1}}{2^{n-2} - 2}\end{aligned}$$

and the procedure's next choice is to set $x_{n-2} = b_{n-2} = 2^{n-3} - 1$. In each iteration $k = 3, \ldots, n-1$ the residual demand thus decreases by an amount of $2^{n-k} - 1$ units. Before iteration k is executed the remaining demand, say D_k, amounts then to $D_k = 2^{n-k+1} - n + k - 1$ and one obtains

$$\frac{f_{n-1}}{D_k} = \frac{2^{n-1}}{2^{n-k+1} - n + k - 1} \geq \frac{f_{n-k+1}}{b_{n-k+1}} = \frac{2^{n-2}}{2^{n-k} - 1} = \frac{2^{n-1}}{2^{n-k+1} - 2}$$

for $k = 3, \ldots, n-1$. Hence, in iterations $k = 3, \ldots, n-1$ the procedure sets $x_{n-k+1} = b_{n-k+1}$ until the residual demand amounts to $D_n = 1$ and x_{n-1} is set to 1 in the last iteration. The objective value of the computed solution is then given by

$$\begin{aligned}Z^{AG} &= \sum_{j=1}^{n} f_j = 1 + \sum_{j=2}^{n-2} 2^{n-2} + 2^{n-1} + n - 3 \\ &= n - 2 + (n-3)2^{n-2} + 2^{n-1} = n - 2 + (n-1)2^{n-2} \\ &= \frac{n-1}{2}\left(2^{n-1} + 2 - \frac{2}{n-1}\right) \geq \frac{n-1}{2}\left(2^{n-1} + 1\right) = \frac{n-1}{2} Z\end{aligned}$$

and the ratio $Z^{AG}/Z \geq \frac{n-1}{2}$ diverges to infinity with increasing problem size n. $\square$

4 Approximation algorithms

Csirik et al. (1991) analyze a greedy-type heuristic of Gens and Levner (1979) for the min-KP and show that this procedure gives a 2-approximation algorithm for this problem. They also provide an improvement to this method having a worst-case performance ratio of 3/2. In the sequel, these methods are adjusted to the case of the SSFCTP.

4.1 A 2-approximation algorithm for the SSFCTP

It is straightforward to adjust the greedy-type method of Gens and Levner (1979) for the min-KP to the case of the SSFCTP: Assume again that the suppliers are sorted according to non-decreasing relative costs $e_j = c_j + f_j/b_j$. The set of all suppliers is then divided into consecutive sets S_q and B_q of so-called "small" and "big" suppliers as indicated below

$$\overbrace{1,\dots,j_1}^{S_1},\ \overbrace{j_1+1,\dots,j_2-1}^{B_1},\ \overbrace{j_2,\dots,j_3}^{S_2},\ \overbrace{j_3+1,\dots,j_4-1}^{B_2},\ \dots,$$
$$,\dots,\ \overbrace{j_l,\dots,j_{l+1}}^{S_l},\ \overbrace{j_{l+1}+1,\dots,j_{l+2}-1}^{B_l},\dots,$$
$$,\dots,\ \overbrace{j_m,\dots,j_{m+1}}^{S_m},\ \overbrace{j_{m+1}+1,\dots,j_{m+2}-1}^{B_m},\ \overbrace{j_{m+2},\dots,n}^{S_{m+1}}.$$

The set S_{m+1} is possibly empty, in which case $j_{m+2} - 1 = n$. No subset of suppliers comprising only small suppliers is able to meet the total demand D; such a solution has always to be completed by including a big supplier. More precisely, we have

$$\sum_{j \in \bigcup_{i=1}^{l} S_j} b_j < D \text{ for all } l = 1, \dots, m+1\,, \tag{15}$$

$$\sum_{j \in \bigcup_{i=1}^{l} S_j} b_j + b_r \geq D \text{ for all } l = 1, \dots, m \text{ and all } r \in B_l\,. \tag{16}$$

The procedure then consists in checking for $l = 1, \dots, m$ all the trial solutions

$$x_j = b_j \text{ for all } j \in S := \bigcup_{i=1}^{l} S_i \quad \text{and} \quad x_r = D - \sum_{j \in S} b_j \text{ for one } r \in B_l$$

and picking the best one. After sorting the suppliers, checking all these solutions can be done in linear time. The method's computational complexity is thus $O(n \log n)$. Moreover the method does also provide a 2-approximation algorithm for the SSFCTP.

Proposition 7. *$Z^{GR} \leq 2Z$ and there exist instances of the SSFCTP such that this bound is tight.*

Proof. Let x^* be an optimal solution to the SSFCTP. Because no set of small suppliers is able to meet the demand, there is at least one big supplier t such that $x_t^* > 0$. Let then $t \in B_q$ for some $q \in \{1, \dots, m\}$ be the supplier with smallest index such that $x_t^* > 0$. Define

$$S = \bigcup_{i=1}^{q} S_i \text{ and } D_S = D - \sum_{j \in S} b_j\,. \tag{17}$$

The heuristic procedure investigates all possibilities of combining supplies from "small suppliers" $j \in \cup_{i=1}^{l} S_i$, $1 \leq l \leq m$, with the supply of one "big supplier" $j \in B_l$. Hence,

$$Z^{GR} \leq \sum_{j \in S} e_j b_j + f_t + c_t D_S\,. \tag{18}$$

Since $x_j^* = 0$ for all big suppliers $j < t$, these suppliers can be removed from consideration. The LP bound denoted by $\tilde{Z}^{LP}$ to this reduced problem is then given by

$$\tilde{Z}^{LP} = \sum_{j \in S} e_j b_j + e_t D_S \, .$$

Hence, we obtain

$$Z^{GR} \le \sum_{j \in S} e_j b_j + f_t + c_t D_S \le \sum_{j \in S} e_j b_j + e_t D_S + f_t = \tilde{Z}^{LP} + f_t \le 2Z \, .$$

To complete the proof, it remains to show that the bound $Z^{GR} \le 2Z$ can be strict. Consider the following series of problem instances that adjusts the example of Csirik et al. (1991) to the case of the SSFCTP: $n = 3$ and

$$\begin{array}{lll} b_1 = 1 \, , & b_2 = D - 2 \, , & b_3 = D - 1 \, , \\ f_1 = 1 \, , & f_2 = D - 2 \, , & f_3 = D - 1 \, , \\ c_1 = 0 \, , & c_2 = 1/(2D) \, , & c_3 = 1/D \, . \end{array}$$

The optimal solution is $x_1 = 1$, $x_2 = 0$, $x_3 = D - 1$, $Z = D + 1 - \frac{1}{D}$. The solution obtained with the above greedy heuristic is $x_1 = 1$, $x_2 = D - 2$, $x_3 = 1$ and has objective function value $Z^{GR} = 2D - \frac{3}{2}$. Hence, Z^{GR}/Z gets arbitrarily close to 2 if D increases. □

It has to be noted that Güntzer and Jungnickel (2000) propose a greedy approach to the min-KP closely resembling that of Csirik et al. (1991) and Gens and Levner (1979), respectively. The only difference is that Güntzer and Jungnickel (2000) investigate additional trial solutions by trying to insert just the first elements of a subset S_l of small items into the knapsack. More or less the same principle can be implemented by removing small items (suppliers) with highest index from the solution produced by the above method as long as this is feasible. Csirik et al. (1991) already hint on this improvement step; the worst-case is however not affected.

4.2 Two 3/2-approximation algorithms for the SSFCTP

Csirik et al. (1991) additionally showed how the above greedy-type method can be improved to 3/2-approximation algorithm for the min-KP. To this end, the method of Gens and Levner is rerun on a reduced problem that results from including in turn each single big item into the knapsack solution. Two slightly different ways of using this idea in case of the SSFCTP can be distinguished.

The first approach consists in the application of the steps listed below.

Step 1: Apply the greedy-type method of Gens and Levner as descibed above. Let Z^{GR} be the solution's objective function value.

Step 2: For each big supplier $i \in B := \bigcup_{l=1}^{m} B_l$ apply the greedy-type method to the modified problem

$$Z_i := \min \sum_{j=1}^{n} c_j x_j + \sum_{j \neq i} f_j y_j \\ \text{s.t.: (2), (3),(4)} \tag{19}$$

and let x^{GR_i} be the solution of objective function value Z^{GR_i} obtained this way for the modified problem.

Step 3: Return the best solution encountered, that is the solution of cost

$$Z^{IGR} = \min\Big\{\min_{i \in B}\{Z^{GR_i} + F_i(x_i^{GR_i}\}, \; Z^{GR}\Big\}\,, \tag{20}$$

where $F_i(x_i) := f_i$ if $x_i > 0$ and $F_i(x_i) = 0$ otherwise.

After the suppliers are sorted in non-decreasing order of the relative costs e_j, restoring this ordering after setting $f_i = 0$ can be done in linear time. The above procedure thus runs in $O(n^2)$ time. The following proposition extends the result of Csirik et al. (1991) obtained for the min-KP to the case of the SSFCTP.

Proposition 8. *$Z^{IGR} \leq \frac{3}{2}Z$. Futhermore, there exist instances of the SSFCTP such that this bound is tight.*

Proof. The proof follows a similar reasoning as that used by Csirik et al. (1991) in their proof of the corresponding worst-case bound for the min-KP. Let x^*, $t \in B_q$, S and D_S be defined as in the proof of Prop. 7. The following two cases are distinguished.

Case 1: $f_t \leq \frac{1}{2}Z$. The proof of Prop. 7 has shown that

$$\begin{aligned} Z^{GR} &\leq \sum_{j \in S} e_j b_j + f_t + c_t D_S \\ &\leq \Big(\sum_{j \in S} e_j b_j + e_t D_S\Big) + f_t \\ &= \tilde{Z}^{LP} + f_t \leq Z + f_t \leq Z + \tfrac{1}{2}Z = \tfrac{3}{2}Z\,, \end{aligned}$$

where $\tilde{Z}^{LP}$ is the LP-bound that results if all big suppliers $j < t$ are not taken into consideration.

Case 2: $f_t > \frac{1}{2}Z$. From (20) it follows that

$$Z^{IGR} \leq Z^{GR_t} + F_t(x_t^{GR_t})\,.$$

From Prop. 7 we have $Z^{GR_t} \leq 2Z_t$. Since x^* is a feasible solution for the modified problem (19), we therewith obtain

$$\begin{aligned} Z^{IGR} &\leq 2Z_t + F_t(x_t^{GR_t}) \\ &\leq 2(Z - f_t) + F_t(x_t^{GR_t}) \\ &\leq 2(Z - f_t) + f_t \leq 2Z - f_t \leq 2Z - \tfrac{1}{2}Z = \tfrac{3}{2}Z\,. \end{aligned}$$

To complete the proof, it remains to show that the bound of $\frac{3}{2}Z$ is tight. An optimal solution to the series of problem instances with $n = 4$, $D = 2M$, $c_1 = c_2 = c_3 = c_4 = 0$, $b_1 = b_2 = M - 1$, $b_3 = b_4 = M$, $f_j = 2b_j$ for $j = 1, \dots, 4$ is given by $x_1^* = x_2^* = 0$, $x_3^* = x_4^* = M$. The objective function value is $Z = 4M$. The greedy-type procedure yields the solution

$$x_1 = x_2 = M - 1\,,\ x_3 = 2 \text{ and } x_4 = 0$$

with objective value $Z^{GR} = 4(M-1) + 2M = 6M - 4$. Finally, the improved greedy procedure gives the solution $x_3 = M$, $x_1 = M - 1$ and $x_2 = 1$. The objective value is

$$Z^{IGR} = Z^{GR_3} + 2M = 4(M-1) + 2M = 6M - 4 = Z^{GR}\,.$$

Hence, $Z^{IGR}/Z = (6M - 4)/(4M) = 3/2 - 1/M$ and this ratio converges to $3/2$ with increasing value of M. $\square$

The second possible way of improving the Gens-Levner type heuristic for the SSFCTP is to fix in turn the supply of each big supplier $i \in B$ to the upper bound b_i and to solve the reduced problem

$$\begin{aligned} Z_i' := \min \ & \sum_{j \neq i} \Big(c_j x_j + f_j y_j \Big) \\ \text{s.t.:} \ & \sum_{j \neq i} x_j = D - b_i\,, \\ & 0 \leq x_j \leq b_j y_j \ \forall\, j \neq i\,, \\ & y_j \in \{0,1\} \quad\ \forall\, j \neq i\,. \end{aligned} \tag{21}$$

by means of the Gens-Levner type heuristic. The approach can be summarized as follows.

Step 1: Apply the greedy-type method of Gens and Levner. Let Z^{GR} be the solution's objective function value.

Step 2: For each big supplier $i \in B := \bigcup_{l=1}^{m} B_l$ apply the greedy-type method to the reduced problem (21) and let $Z^{GR_i'}$ be the objective function value of the solution obtained this way for the reduced problem (21).

Step 3: Return the best solution encountered, that is the solution of cost

$$Z^{IGR'} = \min\Big\{ \min_{i \in B} \{ Z^{GR_i'} + c_i b_i + f_i \},\ Z^{GR} \Big\}\,. \tag{22}$$

Compared to the approach of Prop. 8, the method has the advantage that the ordering of the suppliers need not to be changed when solving the reduced problem. Some further results are, however, required in order to proof that this method also gives a 3/2-approximation algorithm for the SSFCTP.

Lemma 1. Let x^* and $t \in B_q$ with $x_t^* > 0$ be defined as above. As in the proof to Prop. 7, let $\tilde{Z}^{LP}$ be the LP bound that results if the big suppliers $j < t$ are removed from consideration. Then

$$\tilde{Z}^{LP} + f_t\Big(1 - x_t^*/b_t\Big) \leq Z\,. \tag{23}$$

Proof. If $x_t^* = b_t$, the statement is obviously true. With S and D_S defined as in (17), we obtain in the case of $0 < x_t^* < b_t$:

$$\begin{aligned}
Z &\geq c_t x_t^* + f_t + \min\Big\{\sum_{j\neq t} e_j x_j : \sum_{j\neq t} x_j = D - x_t^*\,, 0 \leq x_j \leq b_j\ \forall\, j \neq t\Big\} \\
&\geq c_t x_t^* + f_t + \min\Big\{\sum_{j\neq t} (e_j - e_t) x_j : 0 \leq x_j \leq b_j\ \forall\, j \neq t\Big\} + e_t(D - x_t^*) \\
&= c_t x_t^* + f_t + \sum_{j\neq t} \min\{0, e_j - e_t\} b_j + e_t D - e_t x_t^* \\
&= c_t x_t^* + f_t + \sum_{j\in S} e_j b_j + e_t\Big(D - \sum_{j\in S} b_j\Big) - e_t x_t^* \\
&= c_t x_t^* + f_t + \sum_{j\in S} e_j b_j + e_t D_S - e_t x_t^* \\
&= c_t x_t^* + f_t + \tilde{Z}^{LP} - e_t x_t^* \\
&= \tilde{Z}^{LP} + f_t\Big(1 - x_t^*/b_t\Big)\,.
\end{aligned}$$

The right hand side of the first inequality is the LP bound of the SSFCTP with the additional constraint $x_t = x_t^*$, which shows that the inequality is valid. The second inequality is obtained by further relaxing the demand constraint in a Lagrangean manner with multiplier e_t. □

Prop. 7 and the above lemma directly lead to the following statement.

Lemma 2. Let x^* and $t \in B_q$ be defined as above. If $f_t x_t^*/b_t \leq Z/2$ then $Z^{IGR'} \leq \frac{3}{2}Z$.

Proof. As in the proof to Prop. 7, we have $Z^{GR} \leq \tilde{Z}^{LP} + f_t$. From Lemma 1 we know $\tilde{Z}^{LP} + f_t \leq Z + x_t^* f_t/b_t$. Hence,

$$Z^{GR} \leq Z + x_t^* f_t/b_t \leq Z + \frac{Z}{2} = \frac{3}{2}Z\,,$$

where the second inequality above follows from the assumption. □

The Lemma below uses a different assumption under which the procedure is also a 1.5-approximation method.

Lemma 3. Let x^*, $t \in B_q$, and D_S be defined as before. If

$$c_t(b_t - x_t^*) \leq c_t x_t^* + (f_t/b_t) D_S\,,$$

then $Z^{IGR'} \leq \frac{3}{2}Z$.

Proof. From the way the solution of value $Z^{IGR'}$ is constructed and the fact that the Gens-Levner type heuristic is a 2-approximation algorithm, we have

$$Z^{IGR'} \leq Z^{GR'_t} + c_t b_t + f_t \leq 2Z'_t + c_t b_t + f_t \,.$$

The solution $x_j = x_j^*$ for all $j \neq t$ is a feasible solution to the reduced problem (21), if the equality constraint $\sum_{j \neq t} x_j = D - b_t$ is replaced by $\sum_{j \neq t} x_j \geq D - b_t$. Since the objective function value of the SSFCTP does not decrease with an increasing demand of the sink node, the objective function value of this solution cannot be smaller than Z'_t. Hence,

$$Z^{IGR'} \leq 2(Z - c_t x_t^* - f_t) + c_t b_t + f_t = 2Z - \Big(f_t + c_t(2x_t^* - b_t)\Big) \,.$$

In case of $f_t + c_t(2x_t^* - b_t) \geq Z/2$, the desired result follows immediately. Thus assume $f_t + c_t(2x_t^* - b_t) < Z/2$. As in the proof of Prop. 7, we have

$$Z^{GR} \leq \sum_{j \in S} e_j b_j + f_t + c_t D_S \,.$$

Moreover

$$\sum_{j \in S} e_j b_j + f_t + c_t D_S = f_t + c_t(2x_t^* - b_t) + \sum_{j \in S} e_j b_j + c_t(b_t - x_t^*) - c_t(x_t^* - D_S) \,.$$

Using $f_t + c_t(2x_t^* - b_t) < Z/2$ and $c_t(b_t - x_t^*) \leq c_t x_t^* + (f_t/b_t) D_S$ then gives

$$\begin{aligned} Z^{GR} &\leq Z/2 + \sum_{j \in S} e_j b_j + c_t x_t^* + (f_t/b_t) D_S - c_t(x_t^* - D_S) \\ &= Z/2 + \sum_{j \in S} e_j b_j + \Big((f_t/b_t) + c_t\Big) D_S \\ &= Z/2 + \sum_{j \in S} e_j b_j + e_t D_S = Z/2 + \tilde{Z}^{LP} \leq Z/2 + Z = 3Z/2 \,. \end{aligned}$$

□

The above analyses allows now to state the following proposition.

Proposition 9. $Z^{IGR'} \leq \frac{3}{2} Z$. *Futhermore, there exist instances of the SSFCTP such that this bound is tight.*

Proof. In case of $f_t x_t^*/b_t \leq Z/2$ the result follows from Lemma 2, and in case of $c_t(b_t - x_t^*) \leq c_t x_t^* + (f_t/b_t) D_S$ the result is obtained from Lemma 3. Hence, assume that

$$\frac{f_t}{b_t} x_t^* > Z/2 \quad \text{and} \quad c_t(b_t - 2x_t^*) > \frac{f_t}{b_t} D_S \,.$$

The second condition requires $x_t^* < b_t/2$, and from the first we obtain

$$\frac{Z}{2} < \frac{f_t}{b_t} x_t^* < \frac{f_t}{b_t} \frac{b_t}{2} = \frac{f_t}{2} \quad \Rightarrow \quad Z < f_t \,,$$

which is impossible, since $x_t^* > 0$. The proof is completed by observing that the procedure's solution value also attains its worst case for the problem instance given in the proof to Prop. 8. $\square$

5 Computational comparision

The heuristic algorithms presented in Sect. 3 and 4 were coded in C and run on a 750 MHz Pentium III PC to approximately solve various test problem instances. Optimal solutions for these test problems were obtained using the methods discussed in Klose (2006).

The test problems are divided into two groups. The first group consists of 60 problem instances for each problem size of n = 500, 1000, 5000, and 10000 suppliers. According to the problem generation method proposed by Herer et al. (1996), the problem structure is determined by two parameters.

- The b-ratio $B_r = 100 \cdot D / \sum_j b_j$ estimates the percentage number of suppliers that supply a positive amount in an optimal solution. The computational experiments are based on the four b-ratio values $B_r = 5\,\%$, $10\,\%$, $25\,\%$, and $50\,\%$. (Herer et al. (1996) use the absolute number $B'_r = B_r \cdot n/100$; utilizing the same expected number of employed suppliers for problems that largely differ in size is however not meaningful.)
- The F-ratio $F_r = \overline{f}/(\overline{c}\overline{b})$ is the ratio between the average fixed cost $\overline{f} = \frac{1}{n}\sum_j f_j$ and the average transportation cost $\overline{c}\overline{b}$ per supplier, where $\overline{c} = \frac{1}{n}\sum_j c_j$ and $\overline{b} = \frac{1}{n}\sum_j b_j$. Three different F-ratio values were used, 0.3, 0.6 and 1.

The sink's demand was set to $D = 100000$ and the other problem data were generated randomly such that $f_j \in U[75000, 125000]$, $c_j \in U[8, 12]$, and $b_j \in U[7500, 12500]$, where $U[a_1, a_2]$ denotes a uniformly distributed random number between a_1 and a_2. The fixed costs f_j and the suppliers' capacities b_j were then scaled to meet the desired b-ratio and F-ratio. Furthermore, the capacities as well as the fixed costs were rounded to the nearest integer. Five problem instances were then generated for each problem size and combination of b-ratio and F-ratio.

In the second set of 20 test problems, 5 instances were generated for each of the problem sizes of n = 500, 1000, 5000 and 10000 suppliers. In this case, the suppliers' capacities b_j are integer random numbers from $U[10, 100]$. The sink's demand was set to $D = 0.5\sum_j b_j$ and rounded to the nearest integer. The cost data are then generated such that a positive correlation between the capacity b_j and a supplier j's total cost $C_j := f_j + c_j b_j$ is achieved. To this end, C_j is computed as $C_j = b_j + \beta_j$, where $\beta_j \in [0, b_j]$ is a random integer. The cost C_j is then distributed between fixed and unit cost by setting $f_j = \varphi C_j$ and $c_j = (1 - \varphi)C_j / b_j$ with $\varphi \in U[0.75, 1)$.

A solution x obtained with the greedy and adaptive greedy procedure, respectively, was refined a bit by simply computing the optimal flows on the arcs $k \in J_1 := \{j :$

$x_j > 0\}$. This is easily done by sorting suppliers $j \in J_1$ according to non-decreasing unit costs c_j. Furthermore, we used the already mentioned proposal of Csirik et al. (1991) for slightly improving any candidate solution

$$x_j = b_j \; \forall\, j \in S := \{j_1, \ldots, j_s\} \text{ and } x_r = D - \sum_{j \in S} b_j$$

obtained in the course of the greedy type heuristic of Gens and Levner. To this end, the worst small supplier j_s is removed in turn from the solution and the supply x_r of the big supplier r is increased by x_{j_s} as long as this can feasibly done.

For every single of the 260 test problem instances, the same solution was obtained with the two versions of the 1.5-approximation algorithm for the SSFCTP, Z^{IGR} and $Z^{IGR'}$, respectively. After setting the fixed cost of a big supplier to zero, the supplier is usually pushed a good deal forward in the ordering of suppliers such that he becomes a small supplier with respect to the modified problem (19). Both versions of the 1.5-approximation method will thus set his supply to the upper bound and give identical solutions.

Table 1 compares the solution values obtained with the five heuristic methods on the first group of test problems. The table shows the deviation $1000 \cdot (Z^H - Z)/Z$ of a heuristic solution value Z^H from an optimal value Z in tenth of a percent, averaged over the 5 problem instances per problem size and combination of b-ratio and F-ratio. As can be seen from that table, all methods perform quite well on these types of test problems. On average, the deviation from optimality amounts to 0.10 %, 0.11 %, 0.06 % and 0.02 % for the greedy method (Z^G), adaptive greedy method (Z^{AG}), Gens-Levner heuristic (Z^{GR}) and the two versions of the improved Gens-Levner heuristic (Z^{IGR} and $Z^{IGR'}$, respectively).

Decreasing optimality errors could, on average, be observed with increasing problem size. The deviation from optimality tended also to decline with decreasing values of F_r (expected ratio of fixed cost to transportation cost) and increasing values of B_r (expected percentage number of suppliers employed). Computation times were negligible (below 3 hundreth of a second per problem instance) for all procedures except the two versions of the improved Gens-Levner approach. These two methods call the Gens-Levner heuristic as many times as there are big suppliers, which can be a relatively large number in case of $n = 10000$. On average, the first version of the 1.5-approximation method (Z^{IGR}) required then 0.04, 0.16, 4.45 and 26.77 seconds for problem instances of size $n = 500$, $n = 1000$, $n = 5000$ and $n = 10000$, respectively. The second version of the 1.5-approximation algorithm ($Z^{IGR'}$) was slightly faster. The procedure spent, on average, 0.04, 0.14, 3.57 and 20.61 seconds of CPU time for solving test instances of size $n = 500$, $n = 1000$, $n = 5000$ and $n = 10000$, respectively. Obviously, this large increase in computation times is hardly justified by the relatively small improve in solution quality compared to the simple Gens-Levner method (Z^{GR}).

The results obtained for the second group of test problems are presented in Table 2 (averages over the 5 instances per problem size). Regarding computation times, we

Table 1. Deviation (tenth of a percent) from optimality for group 1 test problems

n	B_r	Z^G			Z^{AG}			Z^{GR}			Z^{IGR} / $Z^{IGR'}$		
		F_r			F_r			F_r			F_r		
		0.3	0.6	1.0	0.3	0.6	1.0	0.3	0.6	1.0	0.3	0.6	1.0
500	5	2.866	3.489	4.514	1.390	7.230	8.962	2.077	1.459	2.940	0.491	0.739	1.534
	10	1.949	4.016	4.353	2.063	3.169	2.928	0.879	1.527	3.753	0.418	0.249	1.388
	25	0.786	0.894	1.909	0.802	1.282	1.913	0.342	0.386	0.718	0.058	0.236	0.200
	50	0.270	0.845	0.975	0.198	0.675	0.703	0.163	0.559	0.352	0.026	0.031	0.061
1000	5	2.159	3.952	4.014	1.635	4.284	2.712	1.481	1.346	3.230	0.421	0.120	1.391
	10	0.355	1.586	1.587	1.267	1.588	2.368	0.073	0.713	0.411	0.015	0.150	0.016
	25	0.194	0.768	0.828	0.119	0.594	0.891	0.171	0.500	0.225	0.043	0.163	0.032
	50	0.161	0.381	0.458	0.180	0.261	0.391	0.172	0.155	0.098	0.015	0.002	0.039
5000	5	0.278	0.673	1.284	0.488	0.461	0.986	0.207	0.348	0.809	0.015	0.006	0.337
	10	0.233	0.237	0.525	0.196	0.403	0.459	0.119	0.143	0.302	0.006	0.009	0.030
	25	0.047	0.110	0.105	0.044	0.078	0.144	0.019	0.030	0.030	0.002	0.000	0.005
	50	0.028	0.039	0.076	0.040	0.046	0.069	0.014	0.020	0.027	0.000	0.000	0.001
10000	5	0.173	0.269	0.570	0.248	0.247	0.392	0.063	0.191	0.472	0.002	0.091	0.178
	10	0.129	0.207	0.220	0.073	0.166	0.159	0.081	0.112	0.130	0.006	0.016	0.047
	25	0.042	0.087	0.061	0.028	0.048	0.059	0.027	0.027	0.029	0.000	0.001	0.000
	50	0.014	0.039	0.035	0.018	0.023	0.015	0.003	0.024	0.019	0.000	0.000	0.000

Table 2
Deviation (tenth of a percent) from optimality for group 2 test problems

n	Z^G	Z^{AG}	Z^{GR}	Z^{IGR} / $Z^{IGR'}$
500	1.734	0.362	0.067	0.038
1000	0.678	0.232	0.036	0.013
5000	0.135	0.036	0.002	0.001
10000	0.067	0.026	0.001	0.000

obtain the same figure as in case of the first group of test problems. Negligible computation times for the first three heuristic procedures, and a substantially larger computational effort required by the two versions of the improved Gens-Levner method. The first 1.5-approximation method took on average 0.03, 0.10, 2.99 and 20.34 seconds of computation time for a problem instance of size $n = 500$, $n = 1000$, $n = 5000$ and $n = 10000$, respectively, whereas the second version of this method required 0.03, 0.10, 2.53 and 16.69 seconds on average. As can be seen from Table 2, the quality of the heuristic solutions improved again with increasing problem size. Although it is generally more difficult to find proven optimal solutions for group 2 than for the group 1 problems, see Klose (2006), the heuristic solutions of Table 2 are even better than those for the group 1 test problems. The two versions of the improved Gens-Levner method gave solutions that were almost optimal. Nevertheless, the relatively small observed improvement in solution quality of these 1.5-approximation methods over the 2-approximation method does hardly justify the large increase in the required computational effort.

6 Conclusions

This paper investigated the worst-case performance of different greedy-type procedures for approximately solving the SSFCTP. It was shown that two popular greedy heuristics might give arbitrarily bad solutions, whereas a 2-approximation method (Gens-Levner heuristic) as well as a 1.5-approximation algorithm (improved Gens-Levner heuristic) could be obtained by adjusting corresponding procedures for the min-knapsack problem. A further direction for future research in this area might be the development of fully polynomial approximation schemes. In addition to the theoretical performance analysis, the different heuristics were also tested empirically on a large set of test problems. The numerical experiments have shown that the improved Gens-Levner heuristic gave almost optimal solutions. Regarding both, the solution quality as well as the required computation time, the simple Gens-Levner type heuristic performed however best.

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